

Precision Holographic Baryons¹

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Abstract. We overview a holographic QCD based on the D4-D8 string theory model, with emphasis on baryons and nucleon-meson interactions thereof. Baryons are realized as holographic images of Skyrmions, but with much qualitative changes. This allows us to derive, without adjustable parameters, couplings of baryons to the entire tower of spin one mesons and also to pseudoscalar mesons. We find some surprisingly good match against empirical values for nucleons, in particular. Tensor couplings to all axial-vectors and iso-singlet vectors all vanish, while, for ρ mesons, tensor couplings are found to be dominant. We close with various cautionary comments and speculations.

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HOLOGRAPHIC QCD

Quantum-Chromodynamics (QCD) in its strongly coupled regime has defied analytical approach for decades. For instance, the large N_c approximation of 't Hooft offered many new insights [1], yet does not let us access the central features of QCD, namely the confinement and the chiral symmetry breaking. In more phenomenological approaches, such as the chiral perturbation theory, we bypass these fundamental questions, and work on phenomenology of low energy hadrons directly. However, this is at the heavy price of introducing many unknown parameters, several for each variety of physical particles, to be fixed by data. At the most practical level, the holography allows us to bridge the gap between QCD and such phenomenological descriptions, by “deriving” an approximate theory of color-singlet objects only, from a large N_c QCD.

Holography originates from a hypothetical duality between a strongly coupled open string theory and weakly coupled closed string theories [2, 3, 4]. In practice, one can really compute the closed string side when it reduces to a tree-level theory involving a handful of massless or nearly massless fields. This invariably includes gravity, since all critical and closed string theory includes massless graviton, and the truncation can be justified only when the open string sides, or Yang-Mills side, is strongly coupled. This latter condition amounts to $\lambda = g_{YM}^2 N_c \gg 1$ in terms of the lowest-lying Yang-Mills sector of the latter [2]. With both N_c and λ large, we replace the strongly interacting quantum theory by a weakly interacting gravity coupled to other light fields that live in some higher dimensional geometry of particular shape. These light fields are supposed to represent gauge-singlet physical particles of the original theory, which are, for QCD, what we call hadrons.

The most obvious question, as far as real QCD goes, is whether we can trust anything

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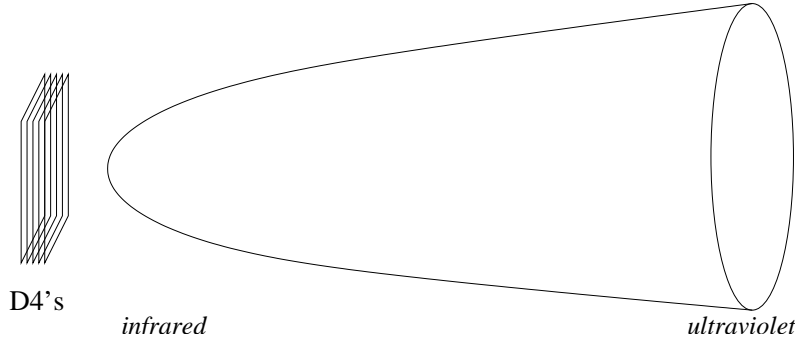


FIGURE 1. The geometry dual to D4 branes is drawn schematically with both Minkowski spacetime and the internal S^4 suppressed. $D4$'s are not actually part of the spacetime manifold. We only exhibit a cigar-like two-dimensional part of the whole ten-dimensional geometry; it encodes how five dimensional supersymmetric Yang-Mills theory on $D4$'s is reduced to a four-dimensional nonsupersymmetric QCD. The geometry is warped, meaning that excitations localized at the bottom (the left end in the figure, denoted as *infrared*) are hierarchically light, so the lightest of physical particles of QCD live there. The infrared curvature scale $\sim M_{KK}$ must be tuned to match a mass scale of QCD.

that is computed in the large N_c and the large λ limit. Real QCD would have $N_c = 3$ while λ is often an extra tunable parameter, which, for our particular model, comes out to be about 17. Whether or not these are big enough to justify the truncation is, at the moment, answerable only by comparing against real world data, so that would be final goal for this talk.

What we can expect and have seen from holographic QCD's are relationships between quantities that are hitherto unexplainable, perhaps except those few quantities already computed by lattice QCD. The earliest such examples involved mass distribution of glueballs [5]. Given a single input data on the lightest glueball, one often finds uncanny match with lattice simulations. The next iteration gave us a series of surprising structures and predictions in the meson sectors, most notably from D4-D8 models of Sakai and Sugimoto [6]. This model not only reproduced the Chiral Lagrangian with couplings to vectors, with only one tunable dimensionless parameter, but it also gave a surprising good prediction on η' mass and a beautiful realization of the vector dominance.

In a similar vein, the holographic baryon in the D4-D8 model [7, 8] is such that we can now discuss, in very precise terms, what are the meson-baryon couplings, how these translate to nucleon-nucleon potential models, and ultimately how such holographic models test against real world data [9, 10, 11]. It is the plan of this talk to overview these development with some emphasis on vector meson-nucleon couplings, which show the true strength of this approach by far.

D4-D8 MODEL AND HOLOGRAPHIC BARYONS

Given the short time allowed for this talk and considering that the most of the audience are nuclear physicists with no or very little exposure to string theory, it is perhaps best to start with a simple and reasonably self-contained prescription of the model, instead of a full-blown derivation.

For four-dimensional QCD with N_f massless flavors, D4-D8 model tells us to solve the following five-dimensional $U(N_f)$ gauge theory at tree-level [6],

$$-\frac{1}{4} \int dx^4 dw \frac{1}{e(w)^2} \text{tr} \mathcal{F}^2 + \frac{N_c}{24\pi^2} \int_{4+1} \omega_5(\mathcal{A}), \quad (1)$$

where the (weak) position-dependent coupling of this *flavor* gauge theory is

$$\frac{1}{e(w)^2} = \frac{\lambda N_c}{108\pi^3} u(w) M_{KK}. \quad (2)$$

and $\omega_5(\mathcal{A})$ is the Chern-Simons 5-form of $U(N_f)$. Almost all the dynamical content of this theory is distilled in the function $u(w)$ that obeys

$$\frac{2}{3} |w| M_{KK} = \int_1^u dy / \sqrt{y^3 - 1}. \quad (3)$$

Note that u ranges from 1 to ∞ while w resides in a finite interval of length $\sim O(1/M_{KK})$. It is clear that M_{KK} sets all the scales in the meson sector while λ serves as additional dimensionless parameter; it is known that $M_{KK} \sim 0.94$ GeV is needed to match the lowest lying ρ meson mass [6].

The origin of this model is in a supersymmetry-broken D4-D8 system in IIA string theory. The large N_c QCD lives in N_c coincident D4-branes, which, like in any holographic model, is replaced by its holographic dual geometry [12]. The 't Hooft-like coupling can be traced to the underlying string theory as

$$\lambda = 2\pi g_s N_c M_{KK} l_s, \quad (4)$$

where g_s is the string coupling and $1/2\pi l_s^2$ is the tension of the fundamental string. By analyzing the gravitational physics in this background, researchers claimed to have found ratios of glueball masses which are consistent with lattice data within 20% or so accuracy [5].

In such a background, N_f D8-branes introduces flavored states [6]. Before taking large N_c limits, open strings connecting D4's and D8's act like quarks, but once D4's are replaced by its holographic dual geometry, the open strings can only connect D8's and D8's, and therefore look like bi-quark mesons. The lowest lying sector of these D8-D8 open string happens to be the five-dimensional $U(N_f)$ gauge theory, from which one can derive a low energy effective action of pions and spin 1 mesons of infinite variety. See section 4 for how four-dimensional mesons arise from this flavor gauge theory.

The fact that we found, in the dual holographic description, a flavor gauge theory is not an accident of this model. The holography, as can be seen from Witten's original prescriptions [4], elevates a continuous global symmetry of the large N field theory in question to a local gauge symmetry in the holographic description. In fact, the universal presence of gravity in the holography can be also understood as such a phenomena, since all relativistic theory possesses the global Poincare symmetry, whose local form is the diffeomorphism invariance. In the present case, $U(N_f)$ can be considered as the local form of $U(N_f)_V$ which is the diagonal subgroup of the usual $U(N_f)_L \times U(N_f)_R$ chiral

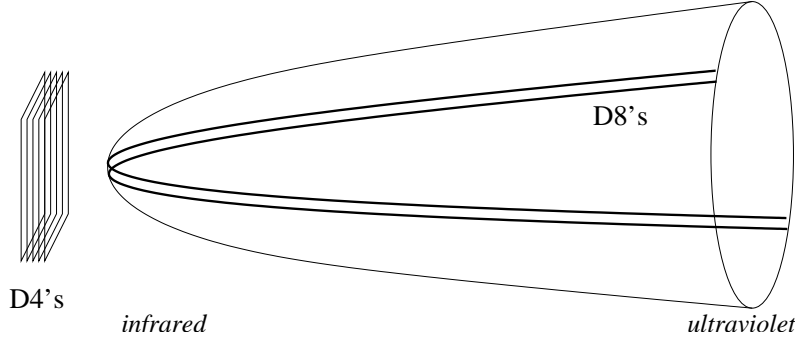


FIGURE 2. Additional D8-branes carry a flavor gauge theory in five dimensions, consisting of w direction, shown here as the radial direction, and the usual four-dimensions of QCD, now shown in this diagram. Holographic baryons are coherent state made from this flavor gauge field, of unit Pontryagin number, and can also be thought of as a holographic image of Skyrmions.

symmetry. The other, axial $U(N_f)_A$ does not elevate to a gauge symmetry, precisely because QCD breaks it dynamically.

This observation is important for us since it also tells us how to find baryons. The baryon carries N_c unit of $U(1)_V \subset U(N_f)_V$ global quantum number, also known as the quark number. Denoting $SU(N_f)$ part of the flavor gauge field by A and its field strength F , one can see that the second term in the five-dimensional flavor theory action includes

$$\frac{N_c}{24\pi^2} \int_{4+1} \omega_5(\mathcal{A}) \simeq \frac{N_c}{8\pi^2} \int_{4+1} \mathcal{A}^{U(1)} \wedge \text{tr} F^2 + \dots, \quad (5)$$

so a soliton that carries a unit Pontryagin number, $\int_4 \text{tr} F^2 = 8\pi^2$, carries a unit baryon number also. In other words, the baryon is a coherent state of \mathcal{A} with a unit topological winding number in $\pi_3(U(N_f)) = \pi_3(SU(N_f))$.

Although similar in spirit to old Skymion picture of the baryon [13], there are qualitative differences. In view of what we will see later when we reduce the theory to four dimensions, the simplest way to describe the connection is to say that the holographic baryon is the Skymion corrected heavily due to couplings to an infinite tower of spin 1 mesons [9]. This difference is important for the phenomenological reasons also, since this new picture allows us to compute how the baryon would couple to all spin 1 mesons, resulting in an infinite number of predictions. Let us proceed and see how this comes about.

AN EFFECTIVE THEORY OF HOLOGRAPHIC BARYONS

Once we identify the baryon as a solitonic object, we must first try to find a solution, quantize it, and then extract its couplings to the flavor gauge field. The approximate form of the soliton was obtained and found to be of very small size [7, 8]

$$\rho_{\text{baryon}} \simeq \frac{(2 \cdot 3^7 \cdot \pi^2 / 5)^{1/4}}{M_{KK} \sqrt{\lambda}}, \quad (6)$$

when $\lambda \gg 1$. (If we had truncated to the pion sector only, upon dimensional reduction, we would have found a Skyrmion of size $\sim 1/M_{KK}$ instead.) It is important to mention here that this size is not what one typically measures in experiment by lepton scattering, since the latter is dominated by the so-called vector dominance and remains finite even as $\lambda \rightarrow \infty$.

Small soliton size is usually a bad sign, since quantum fluctuation of order Compton length might invalidate the classical picture altogether. However, the mass of the baryon scales as $M_{KK}N_c\lambda$, so the Compton wavelength is comfortably smaller than the soliton size. On the other hand, the soliton size is still considerably smaller than $1/M_{KK}$, which as we will see later is the length scale of the holographic mesons in this model.

Combined, these two facts allows a very simple method for finding an effective action for the holographic baryon. The smallness relative to meson mass scale, $\sim M_{KK}$, implies that as long as the meson interaction goes the baryon can be treated as if it is a point-like object. On the other hand, the Compton length's even smaller size implies that we can take the classical shape of the soliton seriously.

For simplicity, we take $N_f = 2$, in which case, assuming odd N_c , the quantization of the soliton gives a Dirac particle in the fundamental representation of $SU(N_f = 2)$ as the lowest lying quantum state. Generalization to higher isospin baryons is straightforward if somewhat more involved. See Ref. [16] for the formulation and Ref. [17] for an application. Introducing a five-dimensional Dirac field \mathcal{B} of isospin 1/2, we arrive at an effective nucleon action with couplings to the flavor gauge field [7, 9],

$$\int d^4x dw \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{2\pi^2\rho_{baryon}^2}{3e^2(w)}\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B} \right]. \quad (7)$$

The position-dependent soliton mass is $m_{\mathcal{B}}(w) = 4\pi^2/e(w)^2$ in the leading $1/N_c$ approximation. The interaction with mesons are captured in two couplings to the gauge field. The first is via the covariant derivative

$$D_m \equiv \partial_m - i(N_c\mathcal{A}_m^{U(1)} + A_m), \quad (8)$$

for which the flavor gauge field \mathcal{A}_m of (1) is decomposed as $\mathcal{A}_m^{U(1)} + A_m$ with traceless 2×2 A_m . This is entirely determined by the conserved quantum numbers of the baryon.

The second, less familiar looking term couples the baryon directly to the $SU(2)$ field strength F . One might mistake that we find this term as a first order correction term, via usual effective theory argument based on operator dimension counting. This is not so, however. This ostensibly higher order term is actually the leading $1/N_c$ coupling to $SU(N_f = 2)$ part of the flavor gauge field. Although we do not elaborate on the derivation here, we emphasize that not only the structure of the operator $\bar{\mathcal{B}}\gamma \cdot F \mathcal{B}$ (which happens to be also unique at dimension six level) but also the coefficient function have been derived rigorously [7]. The derivation relies on the instanton-like nature of the soliton, and shares a mathematical reasoning with the well-known treatment of the Skyrmion by Adkins, Nappi, and Witten [14].

The coefficient function of $\bar{\mathcal{B}}\gamma \cdot F \mathcal{B}$ is actually computable unambiguously only for $w = 0$. Possible deviation would be a multiplicative correction that is unit at $w = 0$. Thankfully, this does not affect large N_c estimates of couplings we consider, because

the baryon wavefunction along w is very tightly localized at $w = 0$ due to the strongly confining nature of $m_{\mathcal{B}}(w)$.

4D PHYSICS

Actions (1) and (7) are meant to be used to generate tree-level Feynman diagrams for mesons and (isospin 1/2) baryons, but they are still in the five-dimensional form. What we need to do at this stage is to reduce the system by performing a Kaluza-Klein reduction along the holographic w -direction. This is exactly what was done to extract glueball masses [5].

For the flavor gauge field, we mode-expand [6, 15] with eigenfunctions, $\psi_{(n)}(w)$,

$$\mathcal{A}_\mu(x; w) = i[U^{-1/2}, \partial_\mu U^{1/2}]/2 + i\{U^{-1/2}, \partial_\mu U^{1/2}\}\psi_0(w) + \sum_n \mathcal{A}_\mu^{(n)}(x)\psi_{(n)}(w) \quad (9)$$

with $U(x) = \exp(i \int_w \mathcal{A})$. The gauge kinetic term in (1) produces two type of terms. The first, from $\psi_{(n)}$'s, contains an infinite tower of vector and axial-vector mesons,

$$\int dx^4 \sum_{n=1}^{\infty} \text{tr} \left\{ \frac{1}{2} \mathcal{F}_{\mu\nu}^{(n)} \mathcal{F}^{(n)\mu\nu} + m_{(n)}^2 \mathcal{A}_\mu^{(n)} \mathcal{A}^{(n)\mu} \right\} + \dots, \quad (10)$$

with the eigenvalue $m_{(n)}^2$ of the KK mode $\psi_{(n)}$, and $\mathcal{F}_{\mu\nu}^{(n)} = \partial_\mu \mathcal{A}_\nu^{(n)} - \partial_\nu \mathcal{A}_\mu^{(n)}$. Because the only length scale in this problem of KK-reduction is $1/M_{KK}$ the masses have the form $m_{(n)}^2 \sim b_n M_{KK}^2$ with a discrete tower of positive numbers b_n 's that start at 0.6 or so. We suppressed interaction terms here but wish to emphasize that all the couplings as well as the masses are unambiguously fixed, once we fix λ and M_{KK} . The meson sector requires $M_{KK} \simeq 0.94$ GeV, to fit the lightest ρ -meson mass, and $\lambda \simeq 17$, to fit f_π to the pion decay constant.

Making contact with four-dimensional spin 1 mesons is done by

$$\mathcal{A}_\mu^{(2k-1)} = \omega_\mu^{(k)} \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} + \rho_\mu^{(k)a} \frac{\tau^a}{2}, \quad (11)$$

for vectors and by

$$\mathcal{A}_\mu^{(2k)} = f_\mu^{(k)} \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} + a_\mu^{(k)a} \frac{\tau^a}{2}, \quad (12)$$

for axial-vectors. Thus, iso-triplets are denoted as ρ 's and a 's while singlets are denoted as ω 's and f 's. Even/odd nature of $\psi_{(n)}(w)$ translates to the usual parity of the corresponding mesons, so ρ 's and ω 's are vectors and a 's and f 's are axial vectors, appearing alternately mass-wise.

The Chiral Lagrangian of Goldstone bosons from the broken chiral symmetry is the other part, from the zero mode ψ_0 ,

$$+ \int dx^4 \left(\frac{f_\pi^2}{4} \text{tr} (U^{-1} \partial_\mu U)^2 + \frac{1}{32e_{Skyme}^2} \text{tr} [U^{-1} \partial_\mu U, U^{-1} \partial_\nu U]^2 \right), \quad (13)$$

with $f_\pi^2 = \lambda N_c M_{KK}^2 / 54 \pi^4$ and $1/e_{\text{Skyme}}^2 \simeq 61 \lambda N_c / 54 \pi^7$. U is related to pions and η' (meaning the $U(1)$ part, regardless of N_f) as

$$U = \exp(\pi i(\eta' + \pi^a \tau_a) / f_\pi) \quad (14)$$

The Chern-Simons terms generate additional interaction term, and among them is the Wess-Zumino-Witten term.

For baryons, let us keep only the nucleons for simplicity, corresponding to the lowest-lying Kaluza-Klein mode, by declaring $\mathcal{B}_\pm(x^\mu, w) = \mathcal{N}_\pm(x^\mu) f_\pm(w)$, with $\gamma_5 \mathcal{N}_\pm = \pm \mathcal{N}_\pm$. Solving the mode equations for the smallest eigenvalue $m_{\mathcal{N}} \simeq m_{\mathcal{B}}(0) + O(M_{KK})$,

$$[-\partial_w^2 \mp \partial_w m_{\mathcal{B}}(w) + (m_{\mathcal{B}}(w))^2] f_\pm(w) = m_{\mathcal{N}}^2 f_\pm(w), \quad (15)$$

and reconstituting \mathcal{N}_\pm into a single four-dimensional nucleon field \mathcal{N} , we find from (7) the following four-dimensional nucleon action [7, 9, 11],

$$\int dx^4 \mathcal{L}_4 = \int dx^4 (-i \bar{\mathcal{N}} \gamma^\mu \partial_\mu \mathcal{N} - i m_{\mathcal{N}} \bar{\mathcal{N}} \mathcal{N} + \dots), \quad (16)$$

where the ellipsis denotes all couplings between nucleons and mesons. These include cubic couplings to chiral Goldstone bosons and axial-vector mesons

$$\frac{1}{2f_\pi} \bar{\mathcal{N}} \gamma^\mu \gamma^5 [g_A \partial_\mu \pi^a \tau_a + g_{\eta'} \partial_\mu \eta'] \mathcal{N} - \frac{1}{2} \bar{\mathcal{N}} \gamma^\mu \gamma^5 \sum_{k \geq 1} [g_f^{(k)} f_\mu^{(k)} + g_a^{(k)} a_\mu^{(k)a} \tau^a] \mathcal{N}, \quad (17)$$

and dimension-four and -five cubic couplings to vector mesons

$$-\frac{1}{2} \bar{\mathcal{N}} \gamma^\mu \sum_{k \geq 1} [g_\omega^{(k)} \omega_\mu^{(k)} + g_\rho^{(k)} \rho_\mu^{(k)a} \tau_a] \mathcal{N} + \frac{1}{2} \bar{\mathcal{N}} \gamma^{\mu\nu} \sum_{k \geq 1} [g_{d\rho}^{(k)} \partial_\mu \rho_\nu^{(k)a} \tau_a] \mathcal{N}. \quad (18)$$

All coupling constants are computed by overlap integrals of type $\int dw f_\pm^* f_\pm \psi_{(n)}$, possibly with a derivative ∂_w acting on $\psi_{(n)}$. In the large λN_c limit, the baryon wavefunctions f_\pm are very sharply peaked at $w = 0$, relative to $\psi_{(n)}$'s, which makes its bilinears act like a delta function. This allows leading large N_c contributions to these couplings be estimated unambiguously.

The last, dimension-five derivative couplings deserve further comments. Note that, among spin 1 mesons, only iso-triplet vectors, denoted as $\rho^{(k)}$, have such derivative-couplings to nucleons. None of axial vectors (iso-singlet f or iso-triplet a), nor any of iso-singlet vectors ω , have a coupling of this type. Interestingly, for ρ mesons, these tensor couplings are dominant over the usual vector couplings. As we will see, for $\rho^{(1)}$ and $\omega^{(1)}$, much of what we find here have been known empirically and used crucially in potential models of nucleons. These are perhaps the most striking results out of D4-D8 holographic baryon picture.

NUMBERS AND COMMENTS

As we noted early on, it is unclear to what extent we should take predictions of such a holographic QCD seriously. A priori, there seems to be more reason not to do so,

especially when we talk about the baryon sector which is very heavy in the large N_c limit. QCD, like any asymptotically-free field theory, comes with a natural scale where the theory become strongly coupled, which in this model is a fraction of M_{KK} . On the other hand, the fact that the holographic QCD is truncated to gravity and its immediate low energy partners implies another scale associated with this truncation, which is in this model nothing but M_{KK} . So, while very low energy sector of such a description may be trustworthy, we appear to have little justification for extending the description beyond 1 GeV. Nevertheless, we find results which mimic experimental findings remarkably well. In fact, it has been explained, based on string theory computations, how the baryon sector of this holographic QCD model can produce as competitive results as the meson sector. We refer the audience to the relevant literature [18, 19].

Probably the most striking and robust prediction of this model is the absence of dimension-five derivative couplings (or tensor couplings) to iso-singlet ω mesons as well as to those of axial-vector mesons [11]

$$g_{d\omega}^{(k)} = 0, \quad g_{df}^{(k)} = 0, \quad g_{da}^{(k)} = 0, \quad (19)$$

as we already noted. This is true as long as we start with (7) and extract the couplings at tree-level.

Remarkably, $g_{d\omega}^{(1)} = 0$ reproduces an empirical fact [20] that, without any theoretical understanding at all, has been used by nucleon-nucleon potential models for decades. In this holographic approach, this vanishing of $g_{d\omega}$'s happens simply because of the instanton-like nature of the baryon, or, in terms of (7), because the derivative couplings exist only for $SU(2)$ part of the flavor gauge field. For axial mesons, which are all heavier than nucleons in real world QCD, empirical values for tensor couplings appear unavailable. It would be most interesting if $g_{df}^{(k)} = 0 = g_{da}^{(k)}$ can also be verified experimentally.

In contrast, $g_{d\rho}^{(k)}$'s do not vanish but are dominant over the dimension-four vector coupling in the large N_c limit [11] as

$$2M_{KK} \times g_{d\rho}^{(k)} \simeq 1.3 \times N_c \times g_{\rho}^{(k)}. \quad (20)$$

After extrapolating to $N_c = 3$ and $\lambda \simeq 17$ and adding a subleading correction, we find

$$2M_{KK} \times g_{d\rho}^{(1)} \simeq 6.0 \times g_{\rho}^{(1)}. \quad (21)$$

If² $m_{\mathcal{N}} \simeq M_{KK}$ as the real world nucleon mass suggests, this would coincide with the empirical ratio between the tensor and the vector coupling, found and used in nucleon potential models [20], which again had no theoretical understanding at all. In particular, this is, from typical low energy effective theory viewpoint, all the more surprising since it tells us that higher dimensional operators can be more important in theories like QCD with its low intrinsic energy scale.

² Unfortunately, prediction of the baryon mass scale relative to the meson one remains the most problematic aspect of this model. The most naive estimate for $m_{\mathcal{N}}$ is too large by a factor of 2, although one must eventually consider summing up all bosonic and fermionic modes around the soliton.

Another set of robust results concern ratios between iso-singlet and iso-triplet couplings. There is a universal relationship of type [9]

$$g_{\omega}^{(k)} / g_{\rho}^{(k)} \simeq N_c + \delta(k) , \quad (22)$$

where the subleading correction $\delta(k)$ turns out to be positive. Again, extrapolating to real QCD regime, we find for $k = 1$

$$g_{\omega}^{(1)} / g_{\rho}^{(1)} \simeq 3 + 0.6 = 3.6 . \quad (23)$$

Extracting ratios like this empirically from experiments is fairly model-dependent, but the ratio is believed to be larger than 3 and numbers around 4 to 5 are typically found [21]. Given the roughness of the approximation involved here, this result is also remarkably good.

Our prediction for g_A , whose experimental value is about 1.27, is [7]

$$g_A = \left(\frac{24}{5\pi^2} \right)^{1/2} \times \frac{N_c}{3} + \dots . \quad (24)$$

With a subleading correction added, the extrapolated value is 0.84 which is about 30% better than Adkins-Nappi-Witten estimate [14] based on the vanilla Skyrmion but worse than an improved Skyrmion dressed by the lowest lying ρ meson with phenomenologically determined couplings [22]. It has been argued that there is a subleading group theoretical correction that shift $N_c \rightarrow N_c + 2$ in (24), upon which we find about 1.3. Validity of this claim remains disputed.

We wish to close with a caveat. Comparisons with experiment must involve direct computation of scattering amplitudes within the model, since, more often than not, empirical values of coupling are somewhat model-dependent. Much work needed to be done here. Nevertheless, it seems that D4-D8 holographic QCD has proven to be very a predictive and successful model of QCD, despite many potential problems in extrapolating to the realistic regime. The baryon sector is particularly difficult to justify, yet produced many robust results that agree with real world observations. This is particularly a pleasant surprise, given the fact that the model has only one dimensionless tunable parameter is.

A holographic model of any known variety cannot hope to compete against various low energy models of mesons and nuclei, for the latter comes with large number of tunable parameters. The main reason for studying holographic QCD model would not be in producing models that are competent in this sense, but rather in advancing our understanding of QCD proper. We hope that numbers presented in this note would eventually prove to be useful guiding lights.

Finally, we must also caution the audience that the model is expected to fail dramatically at very high energy, say, several GeV and beyond, since it does not exhibit the correct Asymptotic Freedom. This is no different from why we do not rely on the Chiral perturbation theory in to study QCD in the perturbative regime. The asymptotic freedom remains among a wish list that must be fulfilled to achieve a true holographic QCD built from reliable and true string theory solutions.

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